Application of Finite Difference Time Domain (FDTD) Technique to Study of Transverse Magnetic Wave Propagation in Two Dimensions System

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Abstract: In this paper, Finite difference time domain technique was applied to find the solutions of Maxwell's curl equations numerically. We calculated the Transverse magnetic (TM_z) wave propagation in two-dimensional (2-D) system in order to describe the propagations of the electric and magnetic waves for the example in Y-branch shape and also structure consists of two elements, each element is constructed as two parallel strips. The results of simulations can describe that the waves propagated and also controlled in a computational domain in 2-D. It was found that the distributions of the TM_z wave can be changed when excited in different phases. The sources of excitations set in the phase and out of phase in two elements to make a comparison between the simulations. Instead of using a metal material such as a copper, we used the perfect electric conductors (PECs) to construct the strips. Therefore, the simulations results indicated that very good distributions were obtained and the waves of propagation controlled between the PECs strips as the electric field component must be set to equal zeros at boundaries in the PECs regions during the calculations. Moreover, this numerical study has demonstrated that the signals appeared identical, equally divided between the elements and propagated in the same phase and amplitude into the upper and lower elements in the Y-branch. The results have proved that the intensities of the TM_z field components can be changed when varied the phases in the calculations.

Keywords: Maxwell's curl equations, finite difference time domain (FDTD) method, two dimensions (2-D)

Introduction:

There are many devices use to generate electromagnetic waves in a free space for examples the coil, dipole antenna, array antenna and waveguide. The different devices design to produce electromagnetic in different distributions to propagate in the space. Therefore, we should study and describe the distributions of the waves and the direction of propagations. This can be done by solving Maxwell's equations analytically if it is possible. When the problem is hard to solve analytically, we require finding the solutions numerically by applying an appropriate numerical technique. In this paper, we have considered the propagation of TM_z waves in the Y-branch shape design and also the waves generated by two elements consisting of two the strips which can be applied to transmit the TM_z waves simultaneously in the space. Therefore, the distributions of the electric and magnetic field components will be generated in a space in many structures in order to make a comparison between the simulations. The aim of this study is to consider the directions of the propagations of the waves when producing in many different designs as constructed in two dimensions such as the Y-branch shape. There are many techniques that can be used such as the method of moments, finite element and finite difference time domain (FDTD) method [1]. The latter will be applied to simulate these designs. Because, the FDTD method is a popular, widely used and efficient method [2, 3] which can be applied to solve several difficult problems in different applications in many fields such as electromagnetism and biomedicine. For the example the method utilizes to compute a specific absorption rate (SAR) [4]. The FDTD method was originated by K. Yee in the paper published in 1966 [5], then developed by many researchers. This method has been developed in order to find the solutions of several complicated problems. As the examples, modelling RF coils and antennas apply in magnetic resonance imaging. Therefore, we can apply the technique in this work to find the solutions of Maxwell's equations numerically. In the FDTD calculations, it is extremely important to include the artificial radiation absorbing boundary condition (ABCs) during the calculation to truncate a gird. The ABCs can solve mainly two problems such as the memory of the computer which is limited and also the ABCs will provide an accurate calculation when reducing the reflections that produce from the boundaries. In this work, there are four edges must be truncated in 2D-FDTD system. Therefore, there are many types of absorbing boundaries conditions (ABCs) have been developed in many applications to overcome this problem such as the truncation boundary condition [6], perfectly matched layer (PML) which was introduced by J. P. Berenger in 1994 [7] and MUR's absorbing boundary condition which was introduced by G. Mur in 1981 [8]. The boundary condition is used to limit a computational domain, which have the advantage to save a computational time. Therefore, the ABCs will be included in all calculations by applying the second order MUR's absorbing boundary condition to truncate the mesh in space which is sufficient as a boundary condition.

Method

This section will describe the basic of the finite difference time domain technique which can be applied to solve Maxwell's curl equations in two-dimensional as the transverse magnetic (TM_z) wave. For this reason the method was employed in order to compute the distributions of the electric and magnetic fields numerically by solving Maxwell's curl equations [9]:

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_o}\nabla \times \mathbf{H} \tag{1.a}$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_o}\nabla \times \mathbf{E} \tag{1.b}$$

Maxwell's equations can be written in two dimensions as the transverse magnetic (TM_z) wave propagates in the *x*-*y* plane as the following [5]:

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_z}{\partial y} \tag{2.a}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial x} \tag{2.b}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{2.c}$$

Where ε_0 and μ_0 are the permittivity and permeability of a free space, respectively.

The equations (2.a and 2.b) show that the temporal derivative of the magnetic field in terms of the spatial derivative of the electric field while the equation (2.c) provides the temporal derivative of the electric field in terms of the spatial derivative of the magnetic fields. By applying the second order accurate of the central finite difference approximation which is expressed as [10]:

$$\frac{\partial F^n(i,j)}{\partial x} = \frac{F^n(i+1/2,j) - F^n(i-1/2,j)}{\delta} + (O\delta^2) \tag{3.a}$$

$$\frac{\partial F^n(i,j)}{\partial t} = \frac{F^{n+1/2}(i,j) - F^{n-1/2}(i,j)}{\delta t} + (O\delta t^2) \tag{3.b}$$

The following notation can indicate a grid point of space as $(i, j) = (i\delta, j\delta)$ and any function of space and time as $F^n(i,j) = F(i\delta, j\delta, n\delta t)$, where δ is the space increment and δt is the time increment.

The central difference approximation in equation (3.a) has the second order accuracy in the space and the order of δ in the term of error $(O\delta^2)$ is two and the same with respect to time as demonstrated in equation (3.b).

The equations (3) can be applied into equation (2) in order to obtain three discrete equations to update the fields the E_z , H_x, H_y components in the space [5]:

$$H_x^{n+\frac{1}{2}}(i, j+\tfrac{1}{2}) = H_x^{n-\frac{1}{2}}(i, j+\tfrac{1}{2}) - \frac{\delta t}{\sqrt{\varepsilon_o\mu_o}\,\delta}\left(E_z^n(i, j+1) - E_z^n(i,j)\right) \tag{4.a}$$

$$H_y^{n+\frac{1}{2}}\left(i+\tfrac{1}{2}, j\right) = H_y^{n-\frac{1}{2}}\left(i+\tfrac{1}{2}, j\right) + \frac{\delta t}{\sqrt{\varepsilon_o\mu_o}\,\delta}\left(E_z^n(i+1, j) - E_z^n(i,j)\right) \tag{4.b}$$

$$E_z^{n+1}(i,j) = E_z^n(i,j) + \frac{\delta t}{\sqrt{\varepsilon_o\mu_o}\,\delta}\left(H_y^{n+\frac{1}{2}}\left(i+\tfrac{1}{2}, j\right) - H_y^{n+\frac{1}{2}}\left(i-\tfrac{1}{2}, j\right)\right) - \left(H_x^{n+\frac{1}{2}}(i, j+\tfrac{1}{2}) - H_x^{n+\frac{1}{2}}(i, j-\tfrac{1}{2})\right) \tag{4.c}$$

The equations (4) can be implemented in a computer MATLAB program language for computing the propagation of a sinusoidal wave in two dimensions. It should be noted from equation (4) that a superscript is used to indicate a time instant while a subscript is used to indicate the coordinates of the vector. The above equations can be normalized by applying the following relation form [11]:

$$\bar{E} = \sqrt{\varepsilon_o/\mu_o}\,E \tag{5}$$

Therefore, the equations (4) can be used to produce three field components (E_z, H_x, H_y) in each cell every time step based on the locations of the electric and magnetic field components as shown in figure 1. The equation 4 is called updating equation every time step. The equations (4.a and 4.b) show that the magnetic field components require the previous value of magnetic components and surrounding the electric components while calculating the electric field requires the previous value of electric field and surrounding of magnetic field components as explained in equation (4.c).

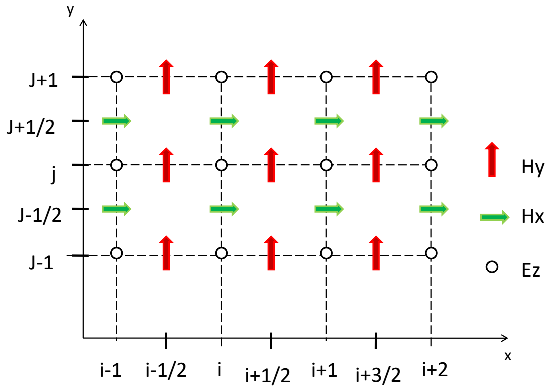

Figure 1: Locations of three components in two-dimensional grid as the transverse magnetic (TM_Z) mode [12].

Including an absorbing boundary condition is very important in the calculations when using the FDTD method to achieve very good electromagnetic distributions as this will lead to obtain the accurate results. The calculation without including the ABCs can affect the distributions as the wave propagated in a space and reached the end of a domain, the wave reflected back to a domain and combined with the incident wave that generated by the source while the calculation with the ABCs, the electromagnetic wave appears to propagate in infinite space as we can call this case is an open domain.

In this study, we calculated a number of examples to make a comparison between the simulations and in all the calculations the grid terminated by the second order MUR's absorbing boundary condition (ABCs). In two-dimensional system, the second order MUR's Boundary condition can be expressed by four differential equations (6) as the following [8]:

At x=0

$$\left(\frac{\partial^2}{\partial x \partial t} - \frac{1}{c}\frac{\partial^2}{\partial t^2} + \frac{c}{2}\frac{\partial^2}{\partial y^2}\right) E_z(0, y, t) = 0, \qquad (6.a)$$

At x=x_{max}

$$\left(\frac{\partial^2}{\partial x \partial t} + \frac{1}{c}\frac{\partial^2}{\partial t^2} - \frac{c}{2}\frac{\partial^2}{\partial y^2}\right) E_z(x_{max}, y, t) = 0, \qquad (6.b)$$

At y=0

$$\left(\frac{\partial^2}{\partial y \partial t} - \frac{1}{c}\frac{\partial^2}{\partial t^2} + \frac{c}{2}\frac{\partial^2}{\partial x^2}\right) E_z(x, 0, t) = 0, \qquad (6.c)$$

At y=y_{max}

$$\left(\frac{\partial^2}{\partial y \partial t} + \frac{1}{c}\frac{\partial^2}{\partial t^2} - \frac{c}{2}\frac{\partial^2}{\partial x^2}\right) E_z(x, y_{max}, t) = 0\ , \qquad (6.d)$$

The boundary condition must be applied in the four edges as at x=0, x=x_{max}, y=0 and y=y_{max} when a uniform mesh is used ($\Delta x = \Delta y = \delta$) [8]. The equation (7) will be implemented in a computer program to calculate the new values at the four edges and the equation (7) is called updating equation in each time step which can be written as [8]:

At x=0

$$E_z^{n+1}(0,j) = -E_z^{n-1}(1,j) + \frac{(c\,\delta t-\delta)}{(c\,\delta t+\delta)}\left(E_z^{n+1}(1,j) + E_z^{n-1}(0,j)\right) + \frac{2\delta}{c\,\delta t+\delta}\left(E_z^{n}(0,j) + E_z^{n}(1,j)\right) + \frac{(c\,\delta t)^2}{2\delta(c\,\delta t+\delta)}\left(E_z^{n}(0,j+1) + E_z^{n}(0,j-1) - 2E_z^{n}(0,j) + E_z^{n}(1,j+1) + E_z^{n}(1,j-1) - 2E_z^{n}(1,j)\right) \qquad (7.a)$$

At x=x_{max},

$$E_z^{n+1}(x_{max},j) = -E_z^{n-1}(x_{max}-1,j) + \frac{(c\,\delta t-\delta)}{(c\,\delta t+\delta)}\left(E_z^{n+1}(x_{max}-1,j) + E_z^{n-1}(x_{max},j)\right) + \frac{2\delta}{c\,\delta t+\delta}\left(E_z^{n}(x_{max},j) + E_z^{n}(x_{max}-1,j)\right) + \frac{(c\,\delta t)^2}{2\delta(c\,\delta t+\delta)}\left(E_z^{n}(x_{max},j+1) - 2E_z^{n}(x_{max},j) + E_z^{n}(x_{max},j-1) + E_z^{n}(x_{max}-1,j+1) - 2E_z^{n}(x_{max}-1,j) - E_z^{n}(x_{max}-1,j-1)\right) \qquad (7.b)$$

At y=0

$$E_z^{n+1}(i,0) = -E_z^{n-1}(i,1) + \frac{(c\,\delta t-\delta)}{(c\,\delta t+\delta)}\left(E_z^{n+1}(i,1) + E_z^{n-1}(i,0)\right) + \frac{2\delta}{c\,\delta t+\delta}\left(E_z^{n}(i,0) + E_z^{n}(i,1)\right) + \frac{(c\,\delta t)^2}{2\delta(c\,\delta t+\delta)}\left(E_z^{n}(i+1,0) + E_z^{n}(i-1,0) - 2E_z^{n}(i,0) + E_z^{n}(i+1,1) + E_z^{n}(i-1,1) - 2E_z^{n}(i,1)\right) \qquad (7.c)$$

At y=y_{max},

$$E_z^{n+1}(i,y_{max}) = -E_z^{n-1}(i,y_{max}-1) + \frac{(c\,\delta t-\delta)}{(c\,\delta t+\delta)}\left(E_z^{n+1}(i,y_{max}-1) + E_z^{n-1}(i,y_{max})\right) + \frac{2\delta}{c\,\delta t+\delta}\left(E_z^{n}(i,y_{max}) + E_z^{n}(i,y_{max}-1)\right) + \frac{(c\,\delta t)^2}{2\delta(c\,\delta t+\delta)}\left(E_z^{n}(i+1,y_{max}) - 2E_z^{n}(i,y_{max}) + E_z^{n}(i-1,y_{max}) + E_z^{n}(i+1,y_{max}-1) - 2E_z^{n}(i,y_{max}-1) + E_z^{n}(i-1,y_{max}-1)\right) \qquad (7.d)$$

The next section will demonstrate the implementations of Maxwell's curl equations and the boundary condition as explained in the results.

Results and discussion

In this section, we will present solutions of Maxwell's equations numerically by using finite difference time domain method. The sinusoidal wave operated at 10 GHz initialled between the strips using a hard source [12] and the electric and magnetic field components were generated in each time step by applying updated equations (4) and the grid terminated by the equation (7) in order to reduce the reflections from the four walls that affect the final simulation results. We have simulated the transverse magnetic waves propagate in two dimensions system. Therefore, in two dimensions 2D-FDTD consists of N_x-by-N_y point lattice [13] in a computational domain. In the following calculations, a grid is set as 150×100 cells in 2D. The cell size equals to ten sampling points per wave length in order to achieve qualitative results.

It can be constructed different shapes to guide the TM_z waves in 2-D as shown in figure 2. The shapes were constructed in the middle of computational domain by using two parallel strips made of the perfect electric conductor (PEC). It means that the electric field component must be set to equal zeros during the calculations. We expect that there are no field components in the locations of the strips and this approach is equivalent to a copper material. There are many examples can be simulated to guide the waves between the strips in different shapes as the example provided in figure 2 (A) was for simulating two parallel strips placed on the left side of a domain, this is the first element. The element has connected with two elements consisting of two strips parallel to each other on the right side of a domain. This structure is called Y–branch shape as the aim of this example is to divide the signals equally with same phase and amplitude in the upper and lower elements. The second example is given for simulating Y-branch as the similar to previous design and in this simulation a small obstacle made of PEC is placed in the upper element as shown in figure 2 (B). The third simulation is that the obstacle made of PEC is placed in the front of the upper element outside the structure as shown in figure 2 (C).

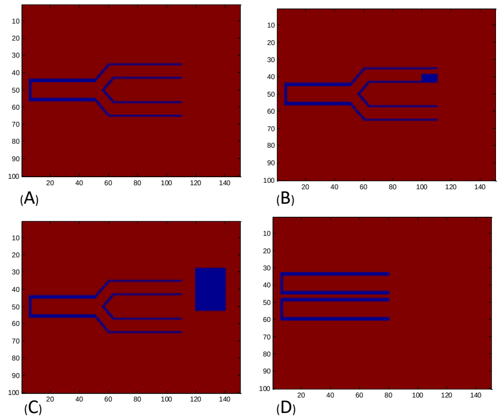

Figure 2: Comparisons between many different designs that were constructed in the computational domain.

The final simulations, two elements consist of two parallel strips are placed very close to each other as shown in figure 2 (D). The purpose of this example is to excite a domain with two elements by two sources operate in phase and out of phase in order to observe the affect of varying the phase on the distributions of the field components. In this case two waves propagate through a medium and combined together. The transverse magnetic components can be generated by a hard source [14] placed between the strips and the computational domain was excited by sinusoidal wave that operated at 10 GHz. In the Y-branch shape design demonstrated in figure 2 (A), we have proved in this example that the electric and magnetic waves propagated between the strips from the left side to right side and then the signal divided to propagate into two elements as shown in figure 3. It should now be noted that the directions of propagations depend on the locations of strips in a domain. By added two identical elements, this structure is called Y-branch and the signals propagated in the upper and lower in Y-branch in the same phase and amplitude as shown in figure 3. The source assignment can be $E_z\,(i, j)$ = source and the source = sin $(2\pi f t)$ as in the first simulation or using the source=sin $(2\pi f t + \theta)$, where θ is 90 degree as setting in the second simulation shown in figure 4. This means that phase difference of a sine waveform is produced. The results of simulations demonstrated that the signal divided equally between two elements as can be clearly observed in figure 5 (A) and also the source operated out of phase 90 degree as shown in figure 5 (B). It can be clearly seen that when figure 3 is compared with figure 4, the intensities of the fields have changed.

Furthermore, we have simulated the structure constructed in figure 2 (B) for making comparison with previous design shown in figure 2 (A) as the obstacle placed between the strips in the upper element. This will affect the distribution of the waves when compared figure 6 and figure 7. The distributions of the waves changed as the PEC caused the wave to reflect back and the intensities of the fields are affected as it can be seen in the front of PEC as shown in figure 6 when comparing the upper and lower elements. The similar results can be obtained when added a small square made of PEC in the front of the upper element as shown in figure 2 (C). Comparing figure 6 with figure 7, it is clearly seen that the obstacle changed the distribution and there is no signal appeared in the locations of obstacles, which means that there is reflection appeared in the image in the front of the obstacle when the results are compared with lower element. Moreover, it can also be simulated two elements parallel to each other as can be seen in figure 2 (D). Each element consists of two parallel strips and this type of configuration can be used to consider the effect of the propagations of the waves when two parallel elements excite a space in the same phase or out phases. Many simulations can be performed to make a comparison between the calculations. For example, consider now the upper element excited and lower element switches off, therefore, the signal will be generated by the upper element in a domain each time step in every cell as shown in figure 8 and the same simulations can be done as the upper element switches off and the signal generated by the lower element as shown in figure 9. Moreover, it can be excited a domain by two elements using two sources operating in the same frequency and in the same phase as shown in figure 10. The wave can be generated by two sources out of phases as 30, 45 and 90 degrees as shown in figure11, figure 12 and figure 13, respectively.

It is interesting to compare the results of simulations that are generated in figure 13 and figure 14. It can be clearly noted that the values of the electric and magnetic fields can be changed by operating the upper and the lower element out of phase by 90 degrees as shown in figure 13 and also figure 14 demonstrates another way of generating the fields as the same simulation was repeated by setting and flipping the sources in the elements out of phase. The previous examples illustrated that the difference appeared in the images due to fact that the waves interfered with each other when the waves propagating simultaneously in the same medium which can be generated the constructive and destructive inferences in the computational domain. It can be obviously observed that there are different distributions generated and appeared in the images compared to the other one when varied the phase in each calculation. It can be proved that varying the phase can lead to change the values of the electric and magnetic components in a domain in each cell. It can now be said that by adding more excitations elements in a space can affect the propagations of TM_z waves and many different distributions can be produced in a space. Therefore, the intensities of the electric and magnetic components can be changed and controlled by varying the phases as demonstrated in the results of the simulations.

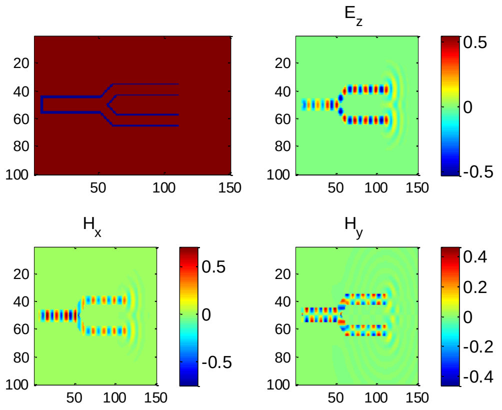

Figure 3: Signals can be generated in the upper and lower parts of Y-branch.

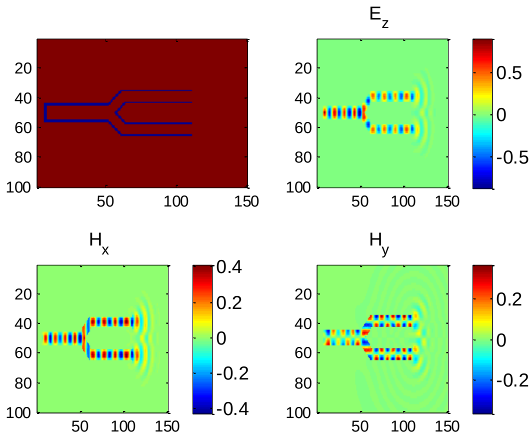

Figure 4: Signals generated 90 degrees out of phase in Y-branch.

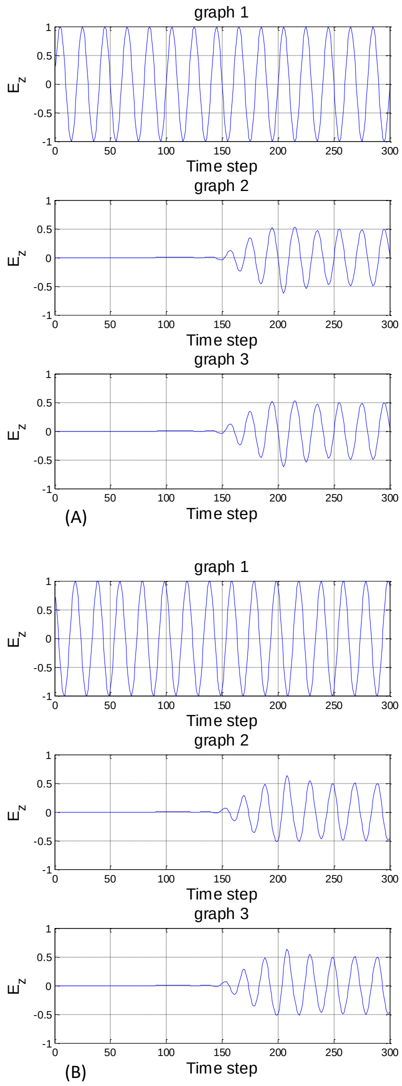

Figure 5: (A) the system reached steady state in Y-branch in the upper and lower parts and (B) the waves generated 90 degrees out of phase in Y-branch.

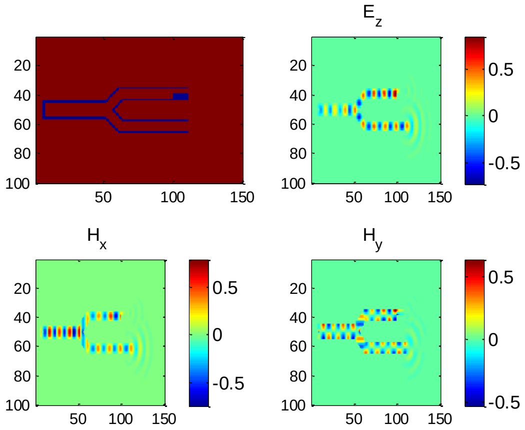

Figure 6: The signals was generated between the strips and an obstacle (PEC) was placed in the upper element.

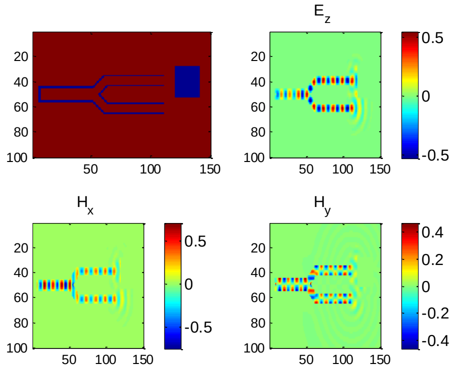

Figure 7: Signals generated between the strips in Y-branch then the signal is striking the obstacle in free space.

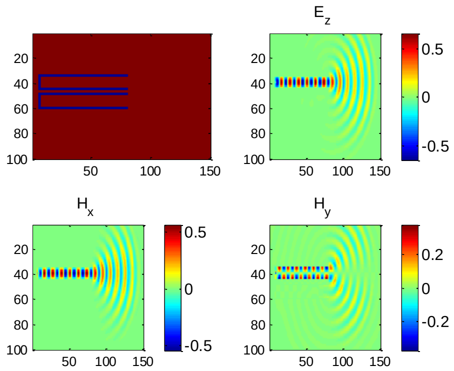

Figure 8: The upper element excited and the lower element switches off.

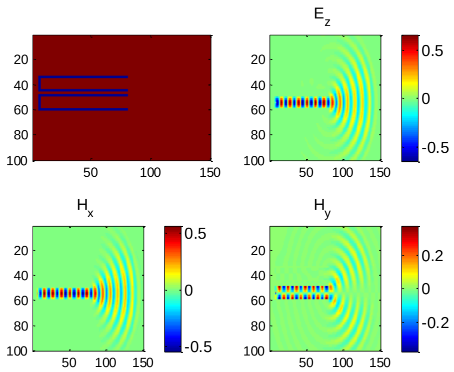

Figure 9: The lower element excited and upper element switches off.

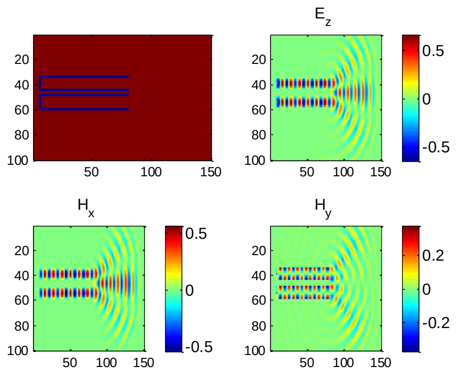

Figure 10: The upper and lower elements are excited a domain by two the sources in the same phase

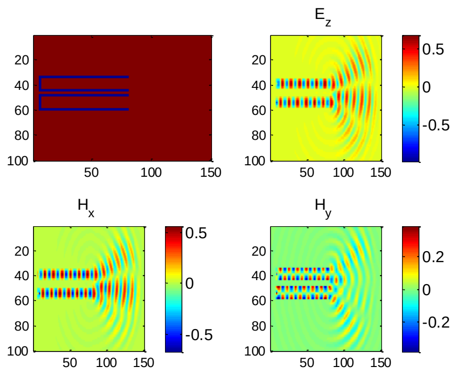

Figure 11: The upper element generated the wave while the lower element generated the wave out of phase 30 degrees.

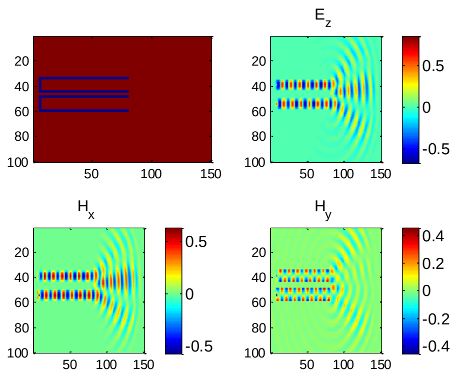

Figure 12: The upper element generated the wave while the lower element generated the wave out of phase 45 degrees.

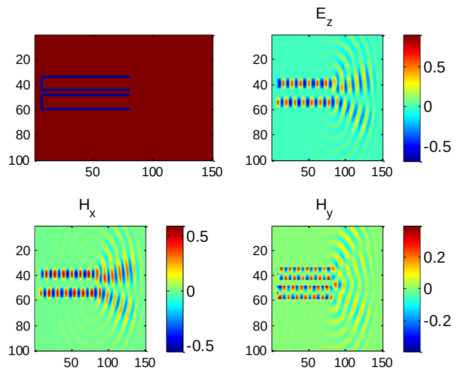

Figure 13: The upper element generated the wave while the lower element generated the wave out of phase 90 degrees.

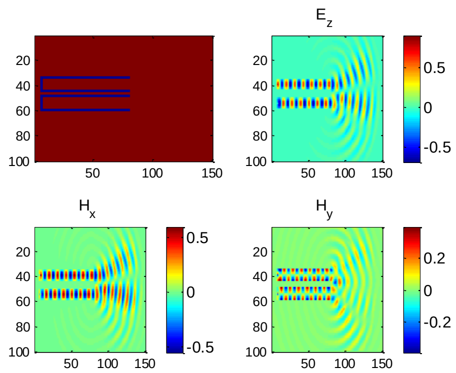

Figure 14: The lower element generated the wave while the upper element generated the wave out of phase 90 degrees.

Conclusions

It can be concluded that one way of describing the propagation of TM_z wave is to solve Maxwell's equations numerically. This can be performed by utilizing the finite difference time domain (FDTD) method. It was found that the method is very good technique to show how 2-D approach can be used to describe the propagation of the waves between the parallel strips in a computation domain. It was noted that the second order's MUR boundary (ABC) condition is appropriate method for using in 2D-FDTD system in order to truncate a computational domain. Therefore, the FDTD method is extremely powerful, efficient and can be successfully employed technique to simulate TM_z wave propagation as explained in this paper.

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تطبيق طريقة الفروق المحددة بالمجال الزمني فى دراسة انتشار الموجة المغناطيسية المستعرضة فى بعدين

الصديق فرحات
جامعة طرابلس، كلية العلوم، قسم الفيزياء

الملخص

في هذه الورقة تم تطبيق طريقة الفروق المحددة بالمجال الزمني لحل معادلات ماكسويل عدديا. تم حساب الموجات المستعرضة في نظام ثنائي الابعاد لوصف انتشار المجال الكهربائي والمغناطيسي مثلا انتشار الموجة في شكل فرع Y حيث يتألف هذا شكل من عنصرين. وايضا محاكاة شكل مركب من عنصرين وكل عنصر مبني على شكل شريطين متوازيين. تم توليد الموجات بين عنصرين وتحكم في الانتشار حسب شكل الشرائط باستخدام الموصل المثالي بدل استخدام مادة موصلة مثل نحاس. تبين من نتائج المحاكاة أن التوزيع الموجي يتغير بتغيير الطور حيث أظهرت هذه الدراسة أن تقسيم الاشارة المنتشرة بنفس السعة و الطور في فرع Y حيث تم توليد توزيع متماثل للموجات في العنصر السفلي و العنصر العلوي. وقد أثبتت الحسابات أن شدة الموجة تعتمد على الطور.